A master equation approach to nonlinear optics

1976 J. Phys. A: Math. Gen. 9 185

(http://iopscience.iop.org/0305-4470/9/1/525)

Corrigendum

A master equation approach to nonlinear optics

McNeil K J and Walls D F 1974 *J. Phys.* A: *Math., Nucl. Gen.* 7 617–31

On page 624 equation (5.10) should read

$$A_{2n} = -\sqrt{\pi(4n-1)}\sum_{l=n}^{\infty}\rho_{2l}(0)\frac{(2l)!}{2^{2l}(l-n)!\Gamma(n+l+\frac{1}{2})}. \tag{5.10}$$

On pages 627, 628 equations (6.4) to (6.6) should be replaced by

$$\bar{\rho}_n(s) = \frac{n!(N_0-n_S^0)!}{(N_0-n)!n_S^0!}\left(\prod_{j=n_S^0}^{n}(s+h_j)\right)^{-1}, \qquad n_S^0 \leqslant n \leqslant N_0 \tag{6.4}$$

where

$$h_j = (j+1)(N_0-j).$$

$\bar{\rho}_n(s) = 0$ otherwise. Let

$$M = \begin{cases} \frac{1}{2}N_0, & N_0 \text{ even} \\ \frac{1}{2}(N_0-1), & N_0 \text{ odd.} \end{cases} \tag{6.5}$$

Then if $n_S^0 \geqslant M$ we obtain repeated factors among the $(s+h_j)$ in (6.4) when $n \geqslant M$. The most convenient way to take these repeated factors into account is to rewrite (6.4):

$$\bar{\rho}_n(s) = \begin{cases} \dfrac{n!(N_0-n_S^0)!}{(N_0-n)!n_S^0!}\left(\displaystyle\prod_{j=n_S^0}^{n}(s+h_j)\right)^{-1}, & n \leqslant M-1 \\ \dfrac{n!(N_0-n_S^0)!}{(N_0-n)!n_S^0!}\left(\displaystyle\prod_{j=n_S^0}^{M-1}(s+h_j)\prod_{k=M}^{n}(s+h_k)\right)^{-1}, & n \geqslant M \text{ and } n_S^0 \leqslant M. \end{cases} \tag{6.6}$$

Inversion of (6.6) (using convolutions where necessary) yields

$$\rho_n(\tau) = \begin{cases} \dfrac{n!(N_0-n_S^0)!}{(N_0-n)!n_S^0!}\displaystyle\sum_{j=n_S^0}^{n} A_j^{n_S^0,n}\exp(-h_j\tau), & n \leqslant M-1 \\ \dfrac{n!(N_0-n_S^0)!}{(N_0-n)!n_S^0!}\displaystyle\sum_{j=n_S^0}^{M-1}\sum_{k=M}^{n} A_j^{n_S^0,M-1}A_k^{M,n} & \\ \times\{(1-\delta_{h_j,h_k})[\exp(-h_j\tau)-\exp(-h_k\tau)]/(h_k-h_j) & n \geqslant M \text{ and } n_S^0 \leqslant M \\ +\delta_{h_j,h_k}\tau\exp(-h_j\tau)\} & \end{cases} \tag{6.7}$$

where the $A_j^{m,n}$ are the appropriate coefficients in the partial fraction expansions of the denominator products in (6.6), ie

$$A_j^{m,n} = \left(\prod_{\substack{i=m\\ i\neq j}}^{n}(h_i-h_j)\right)^{-1} \tag{6.8}$$

On page 629 equation (7.2) should read

$$\frac{d\langle n^2\rangle}{d\tau} = -4\langle n^3\rangle + 8\langle n^2\rangle - 4\langle n\rangle. \tag{7.2}$$

Equation (7.3) should read

$$\langle n(\tau)\rangle \simeq [\langle n(0)\rangle + (1 - \langle n(0)\rangle)\, e^{\tau}]^{-1} \langle n(0)\rangle. \tag{7.3}$$

The authors wish to thank Professor R Loudon and H D Simaan for very helpful correspondence on these points.